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## CREEP AND RELAXATION STRESS OF DISPERSED MIX IN STATIC LOADING CONDITIONS

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A diversity of dispersed systems, their applied significance define the necessity of deep studying their properties and developing methods of physical-and-mechanical controlling their properties at the different stages of technological processes of dispersed systems treating. The abovementioned can be completely related to the process of forming.

The main problem is building a mathematical model of deforming a dispersed material layer. The fact that the problem has not been solved yet till nowadays is connected with the presence of a number of state additional parameters and with an essential complexity of state equations.

As a result of complicated rheological properties of even such an ideal medium as dry sand is, researchers cannot still find adequate determining equations. The presence of resin in the mix leads to demonstrating viscous properties due to the surface effects in dispersed materials. Their presence in the mix not only changes the parameters of mechanical properties in the quantity terms, but it leads to visible viscous effects. The presence of an air layer in the steam conditions special abnormal effects. In this connection alongside with theoretical building there is necessary to pay special attention to experimental revealing the additional parameters of the dispersed layer state. The problem is complicated by the fact that forming is carried out under the action of the gas flow pressure. This requires a purposeful studying of the process gas mechanics relates, first of all, to the loading pulses effect on the molding sand strength. The analysis of cyclic effects on the mix is important both in connection with studying the pulses of different power effect on the elastic modules of the dispersed medium and from the point of view of the changing of steam volume, inner friction angle, and shear strength.

The nonlinear strain of the dispersed medium is conditioned, mainly, by the various character of elastic, viscous and plastic strain dependence on strain.

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Elastic strain depends on stress linearly, and plastic and viscous strains (depend on the binder viscosity, including the value and speed of the dispersed medium heating) are connected with stress nonlinearly. The component of the strain viscous part is determined from the differential equation  $\sigma = 3 \cdot \eta_c \cdot \dot{\epsilon}_n$  taking into consideration  $\eta_c = \eta_0 \cdot (1 + q \cdot t)$  [1], where  $\sigma$  is stress;  $\eta_c$  is viscosity coefficient;  $\dot{\epsilon}_n$  is deformation speed;  $\eta_0$  is viscosity coefficient at the moment of sand-and-resin mix viscosity changing.

$$\sigma = \eta_0 \cdot (1 + q \cdot t) \cdot \frac{d\epsilon}{dt}$$

As the load in the mix is static and the loading on the whole is performed with a constant speed, let's take the boundary condition  $\sigma = const$ . Then, dividing the variables and integrating each part, we'll obtain a viscous component:

$$\epsilon_n = \frac{\sigma}{\eta_0 \cdot q} \cdot \ln(1 + q \cdot t)$$

A plastic deformation component will be determined from  $\sigma = 3 \cdot \lambda \cdot \epsilon_n$ . By the similar method:

$$\epsilon_n = \frac{\sigma}{\lambda_0 \cdot w} \cdot \ln(1 + w \cdot t),$$

where  $\lambda_0$  is plasticity module at the time moment  $t = 0$ ;  $w$  is a coefficient of correction for the sand-and-resin mix plasticity.

In such a case the law of the mix deforming under a static load [1] can be presented by dependence (1):

$$\epsilon = \frac{\sigma}{E} + \frac{\sigma}{\lambda_0 \cdot w} \cdot \ln(1 + w \cdot t) + \frac{\sigma}{\eta_0 \cdot q} \cdot \ln(1 + q \cdot t) \quad (1)$$

where  $\epsilon$  is mix complete deformation;  $\sigma$  is mix stress under the load action;  $E$  is elasticity module.

Let's designate  $\lambda_0 \cdot w = E / a$ ,  $\eta_0 \cdot q = E / b$ , where  $a$ ,  $b$  are proportionality coefficients. Then:

$$\epsilon = \frac{\sigma}{E} + a \cdot \frac{\sigma}{E} \cdot \ln(1 + w \cdot t) + b \cdot \frac{\sigma}{E} \cdot \ln(1 + q \cdot t) = \frac{\sigma \cdot [1 + a \cdot \ln(1 + w \cdot t) + b \cdot \ln(1 + q \cdot t)]}{E}$$

Let's designate  $\sigma \cdot [1 + a \cdot \ln(1 + w \cdot t) + b \cdot \ln(1 + q \cdot t)] = \sigma_{\phi}$ , where  $\sigma_{\phi}$  is the stress which should be applied to the sand-and-resin dispersed medium with the elasticity module  $E$  in the case of linear deformation so that to cause a defor-

mation equal to one obtained from the real stress  $\sigma$  of the dispersed medium, which is deformed nonlinearly with elasticity module  $E$ , plasticity module  $\lambda_0$  and viscosity coefficient  $\eta_0$ .

In the unbalanced state the stress is equalized by elastic and viscous-and-plastic resistance. From here:

$$\sigma_y = \sigma [1 + a \cdot \ln(1 + w \cdot t) + b \cdot \ln(1 + q \cdot t)] = E \cdot \varepsilon + E \cdot \tau \cdot \frac{d\varepsilon}{dt}$$

where  $\varepsilon$  is unbalanced deformation;  $E \cdot \tau$  is linear viscosity.

$$\sigma = \frac{E}{1 + a \cdot \ln(1 + w \cdot t) + b \cdot \ln(1 + q \cdot t)} \left( \varepsilon + \tau \cdot \frac{d\varepsilon}{dt} \right)$$

At the starting moment before the loading action upon the mix ( $t = 0$ ):

$$\sigma_0 = \frac{E_0}{1} \cdot \left( \varepsilon_0 + \tau_0 \cdot \frac{d\varepsilon_0}{dt} \right),$$

where  $E_0$ ,  $\varepsilon_0$ ,  $\tau_0$  are respectively elasticity module, mix deformation, and the creep period at the time moment  $t = 0$ .

Stress in the mix with the applied load:

$$\sigma_y = \sigma [1 + a_1 \cdot \ln(1 + w \cdot t) + b_1 \cdot \ln(1 + q \cdot t)] = E_1 \cdot \left( \varepsilon_1 + \tau_1 \cdot \frac{d\varepsilon_1}{dt} \right)$$

Thus, we can write down:

$$E_0 \varepsilon_0 + E_0 \tau_0 \frac{d\varepsilon_0}{dt} = \sigma_0; E_1 \varepsilon_1 + E_1 \tau_1 \frac{d\varepsilon_1}{dt} = \sigma_1 [1 + a_1 \cdot \ln(1 + w \cdot t) + b_1 \cdot \ln(1 + q \cdot t)]$$

where  $E_1$ ,  $\varepsilon_1$ ,  $\tau_1$  are respectively elasticity module, mix deformation, and the creep period at the time moment  $t = t_1$ .

To simplify let's change  $(1 + q \cdot t) = C$ ,  $(1 + w \cdot t) = D$ .

The solution will be carried out similarly, as it's shown in [1]. As a result of the system combined solution relatively to  $\varepsilon$ , where  $\varepsilon = \varepsilon_0 + \varepsilon_1$ , ( $\varepsilon_0$ ,  $\varepsilon_1$  – starting and current values of deformation respectively, we'll obtain the following differential equation (2):

$$\begin{aligned} E_0 \cdot E_1 \cdot \tau_0 \cdot \tau_1 \cdot \frac{d^2 \varepsilon}{dt^2} + (E_0 \cdot E_1 \cdot \tau_0 + E_0 \cdot E_1 \cdot \tau_1) \cdot \frac{d\varepsilon}{dt} + E_0 \cdot E_1 \cdot \varepsilon = \\ = [E_0 \cdot \tau_0 + E_1 \cdot \tau_1 + E_0 \cdot \tau_0 \cdot (1 + a_1 \cdot \ln C + b_1 \cdot \ln D) + E_1 \cdot \tau_1] \cdot \frac{d\sigma}{dt} + \\ + [E_0 + E_1 + E_0 \cdot (1 + a_1 \cdot \ln C + b_1 \cdot \ln D) + E_1] \cdot \sigma \end{aligned} \quad (2)$$

Let's consider the mix deforming when loaded with a constant speed. The

$\sigma = const$ ,  $d\sigma / dt = 0$ ,  $\sigma = \sigma \cdot t$ , where  $t$  is time. In such a case equation will take the form:

$$\begin{aligned} E_0 \cdot E_1 \cdot \tau_0 \cdot \tau_1 \cdot \frac{d^2 \varepsilon}{dt^2} + E_0 \cdot E_1 \cdot (\tau_0 + \tau_1) \cdot \frac{d\varepsilon}{dt} + E_0 \cdot E_1 \cdot \varepsilon = \\ = [E_0 \cdot \tau_0 + E_1 \cdot \tau_1 + E_0 \cdot \tau_0 \cdot (1 + a_1 \cdot \ln C + b_1 \cdot \ln D) + E_1 \cdot \tau_1] \cdot \sigma + \\ + [E_0 + E_1 + E_0 \cdot (1 + a_1 \cdot \ln C + b_1 \cdot \ln D) + E_1] \cdot \sigma \cdot t \end{aligned} \quad (3)$$

The general solution of this equation is the function [1]:

$$\varepsilon = A_1 e^{\alpha_1 t} + A_2 e^{\alpha_2 t} + B_1 t + B_2 \quad (4)$$

where  $\alpha_1$ ,  $\alpha_2$  are the roots of the equation  $\tau_0 \cdot \tau_1 \cdot \alpha^2 + (\tau_0 + \tau_1) \cdot \alpha + 1 = 0$ , from where we find:

$$\alpha_1 = -\frac{1}{\tau_0}, \alpha_2 = -\frac{1}{\tau_1} \quad (5)$$

$A_1$ ,  $A_2$  are integration constants.

$$B_1 = \left[ \frac{\tau_0}{E_1} + 2 \cdot \frac{\tau_1}{E_0} + \frac{\tau_0}{E_1} \cdot (1 + a_1 \cdot \ln C + b_1 \cdot \ln D) \right] \cdot \sigma,$$

$$B_2 = \left[ \frac{1}{E_1} + \frac{2}{E_0} + \frac{1}{E_1} \cdot (1 + a_1 \cdot \ln C + b_1 \cdot \ln D) \right] \cdot \sigma.$$

To determine integration constants we take into consideration that at  $t = 0$

$$\varepsilon = \frac{d\varepsilon}{dt} = 0.$$

From equation (4) it's seen that at  $t = 0$ :

$$A_1 + A_2 + B_2 = 0 \quad (6)$$

From the condition  $\frac{d\varepsilon}{dt} = 0$

$$A_1 \cdot \alpha_1 + A_2 \cdot \alpha_2 + B_1 = 0 \quad (7)$$

We'll solve (6) and (7) jointly and find:

$$A_1 = \frac{B_2 \cdot \alpha_2 - B_1}{\alpha_1 - \alpha_2}, A_2 = \frac{B_1 - B_2 \cdot \alpha_1}{\alpha_1 - \alpha_2}$$

Then we substitute instead of  $\alpha_1$  and  $\alpha_2$  their values from (5) and obtain:

$$A_1 = \frac{\tau_0 \cdot (B_2 + B_1 \cdot \tau_1)}{\tau_1 - \tau_0}, A_2 = \frac{\tau_1 \cdot (B_1 \cdot \tau_0 + B_2)}{\tau_0 - \tau_1}$$

Excluding from (4)  $\alpha_1$ ,  $\alpha_2$ ,  $A_1$ ,  $A_2$  based on the values obtained for them, we'll obtain the following dependence:

$$\varepsilon = \frac{\tau_0 \cdot (B_2 + B_1 \cdot \tau_1)}{\tau_1 - \tau_0} \cdot e^{-\frac{t}{\tau_0}} + \frac{\tau_1 \cdot (B_1 \cdot \tau_0 + B_2)}{\tau_0 - \tau_1} \cdot e^{-\frac{t}{\tau_1}} + B_1 t + B_2 \quad (8)$$

$t = \frac{\sigma}{\sigma}$ . At  $a_1 = a_0 := b_1 = b_0 = 0$ :

$$B_1 = \left[ \frac{\tau_0}{E_1} + 2 \cdot \frac{\tau_1}{E_0} + \frac{\tau_0}{E_1} \right] \cdot \sigma; B_2 = \left[ \frac{1}{E_1} + \frac{2}{E_0} + \frac{1}{E_1} \right] \cdot \sigma.$$

in substituting these private values into (8) we obtain dependence (9):

$$\varepsilon = \frac{\tau_0 \cdot \left[ \left( 2 \cdot \frac{\tau_1}{E_0} + 2 \cdot \frac{\tau_0}{E_1} \right) \cdot \sigma \cdot \tau_1 + \left( \frac{2}{E_1} + \frac{2}{E_0} \right) \cdot \sigma \right] \cdot e^{-\frac{t}{\tau_0}} + \tau_1 \cdot \left[ \left( 2 \cdot \frac{\tau_1}{E_0} + 2 \cdot \frac{\tau_0}{E_1} \right) \cdot \sigma \cdot \tau_0 + \left( \frac{2}{E_1} + \frac{2}{E_0} \right) \cdot \sigma \right] \cdot e^{-\frac{t}{\tau_1}} + \left( 2 \cdot \frac{\tau_0}{E_1} + 2 \cdot \frac{\tau_1}{E_0} \right) \cdot \sigma \cdot t + \left( 2 \cdot \frac{1}{E_1} + 2 \cdot \frac{1}{E_0} \right) \cdot \sigma \quad (9)$$

s, we can determine a dispersed sand-and-resin mix deformation dependence on the stress speed and time taking into consideration elastic, viscous, and components of deformation. Comparing theoretical and practical results shown in Figure 1.

we determine the value of stresses in the sand-and-resin mix creep. As  $\sigma = 0$ , then  $\sigma = 0$ , and taking into consideration  $\tau_0 = 0$ , we'll obtain from (9):

$$\varepsilon = \left( -\frac{2 \cdot \tau_1}{E_0} \cdot e^{-\frac{t}{\tau_1}} \right) \cdot \sigma + \left( \frac{2 \cdot \tau_1}{E_0} \right) \cdot \sigma = \sigma \cdot \left[ \frac{2 \cdot \tau_1}{E_0} \cdot (1 - e^{-\frac{t}{\tau_1}}) \right] \quad (10)$$

dependence (10) is the equation of sand-and-resin mix creep under the static load applied to the mix.

we let's determine relaxation stress. In equation (3) let  $\varepsilon = \varepsilon_p = const$ :

$\dot{\sigma} = 0$  and further

$$E_0 \cdot \varepsilon_p = [E_0 \cdot \tau_0 + E_1 \cdot \tau_1 + E_0 \cdot \tau_0 \cdot (1 + a_1 \cdot \ln C + b_1 \cdot \ln D + E_1 \cdot \tau_1)] \cdot \frac{d\sigma}{dt} + [E_0 + E_1 + E_0 \cdot (1 + a_1 \cdot \ln C + b_1 \cdot \ln D) + E_1] \cdot \sigma.$$

from here:

$$\frac{d\sigma}{dt} = \frac{E_0 \cdot E_1 \cdot \varepsilon_p - [E_0 + 2 \cdot E_1 + E_0 \cdot (1 + a_1 \cdot \ln C + b_1 \cdot \ln D)] \cdot \sigma}{E_0 \cdot \tau_0 + E_1 \cdot \tau_1 + E_0 \cdot \tau_0 \cdot (1 + a_1 \cdot \ln C + b_1 \cdot \ln D + E_1 \cdot \tau_1)} \quad (11)$$

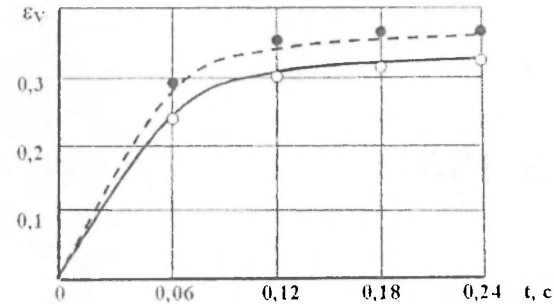


Figure 1. Comparing theoretical and experimental data of sand-and-resin mix strain distribution in time

At  $\frac{d\sigma}{dt} = 0$   $\sigma = \sigma_p$ , where  $\sigma_p$  is a balanced value after relaxation. From (11) we'll obtain  $E_0 \cdot E_1 \cdot \varepsilon_p - [E_0 + 2 \cdot E_1 + E_0 \cdot (1 + a_1 \cdot \ln C + b_1 \cdot \ln D)] \cdot \sigma_p = 0$ , from where  $[E_0 + 2 \cdot E_1 + E_0 \cdot (1 + a_1 \cdot \ln C + b_1 \cdot \ln D)] \cdot \sigma_p = E_0 \cdot E_1 \cdot \varepsilon_p$ .

For a private case, when the mix is deformed linearly,  $a_1 = b_1 = 0$  and then  $[2 \cdot E_0 + 2 \cdot E_1] \cdot \sigma_p = E_0 \cdot E_1 \cdot \varepsilon_p$ .

From the equality obtained we can determine the final relaxation stress with the help of formula (12)

$$\sigma_p = \frac{E_0 \cdot E_1 \cdot \varepsilon_p}{2 \cdot (E_0 + E_1)} \quad (12)$$

Dependence (12) is the equation of the sand-and-resin mix relaxation under the static load applied to the mix.

Thus, the nonlinear deformation of dispersed sand-and-resin medium is conditioned by the various character of elastic, viscous and plastic deformation dependence on stress. There are obtained the equations of relaxation and creep for the dispersed sand-and-resin mix under static loading.

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